Optical cavity QED

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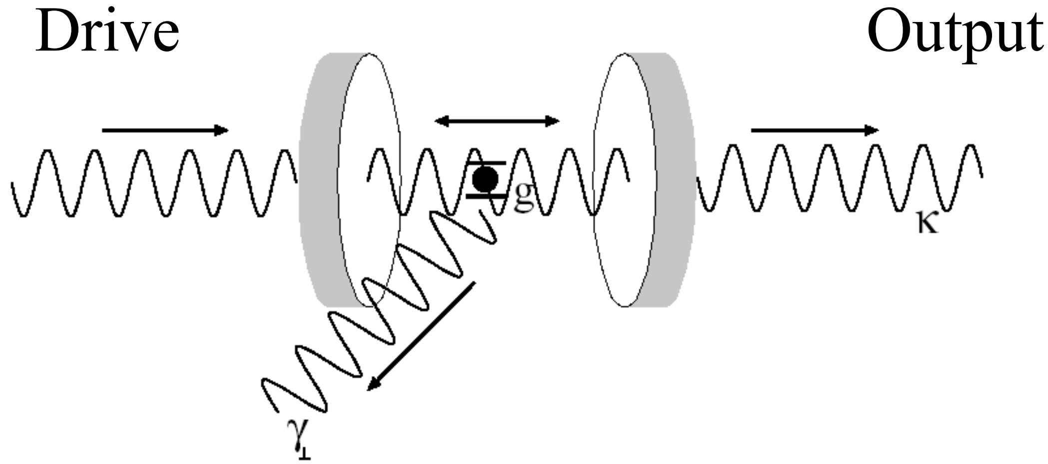
Coupled atoms and cavities:



Collection of N Two level atoms coupled to a single mode of the electromagnetic field. This is a far from equilibrium system. Driven with dissipation (atoms γ, cavity κ).

Microwaves → Micromaser → Cavity QED

Visible light → Optical Bistability → Cavity QED

Dipole coupling between the atom and the cavity.

$$g = \frac{d \cdot E_v}{\hbar}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordon coefficients from the angular part of the integral. It is of the order of a few times a_0 (Bohr radius) times the electron charge e

$$\vec{d} = e\left\langle 5S_{1/2}\middle|\vec{r}\middle|5P_{3/2}\right\rangle$$

The field of one photon in a cavity with Volume V_{eff} is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

The electric field squared is an energy density.

Single atom Cooperativity (measures the effect of one atom):

$$C_1 = \frac{g^2}{\kappa\gamma}$$

Saturation photon number (measures the effect of one photon):

$$n_0 = \frac{\gamma^2}{3g^2}$$

Cooperativity (for N atoms): is the ratio of the atomic losses to the cavity losses or also can be read as the ratio between the good coupling (g) and the dissipation (κ,γ).

$$C = C_1 N = \frac{\alpha_0 l}{T}$$

Are the two definitions equivalent?

$$C_1 = \frac{g^2}{\kappa\gamma}; \quad g^2 = \frac{1}{\hbar^2}\frac{d^2\hbar\omega}{2\varepsilon_0 V} = \frac{d^2\omega}{2\hbar\varepsilon_0 A l}$$

$$\kappa = \frac{c}{2l}T; \quad \gamma = \frac{4}{3}\frac{\omega^3}{c^2}\frac{d^2}{4\pi\varepsilon_0\hbar c}$$

$$C = C_1 N = \frac{\alpha_0 l}{T} = \frac{\sigma\rho l}{T} = \frac{3}{2\pi}\frac{\lambda^2}{A}\frac{N}{T}$$

RATIO OF TWO AREAS times some ENHANCEMENT

Cavity QED systems

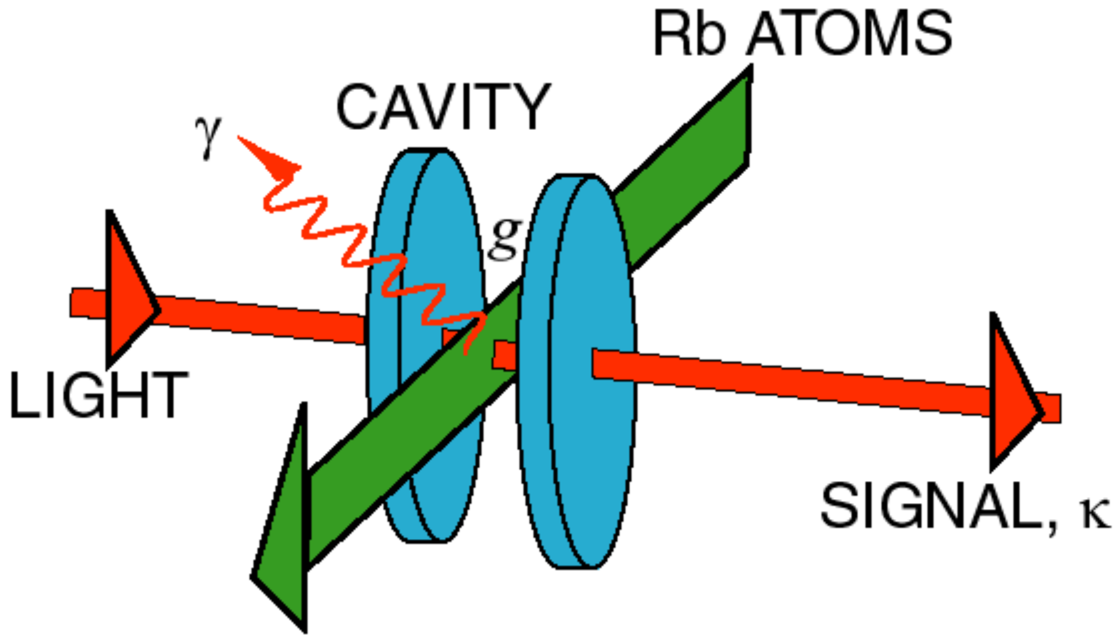
Optical free space

Optical matter

Microwave free space

Microwave matter

Typical system for optical experiments.



$$\frac{g}{2\pi} = 4.7 \text{ MHz up to } 12 \text{ MHz}$$

$$\frac{\kappa}{2\pi} = 3.6 \text{ MHz}$$

$$\frac{\gamma}{2\pi} = 6.0 \text{ MHz}$$

Optical Free Space (resonant)

- A saturable absorber (an atom) has an absorption coefficient which is a non-linear function of the intensity I:

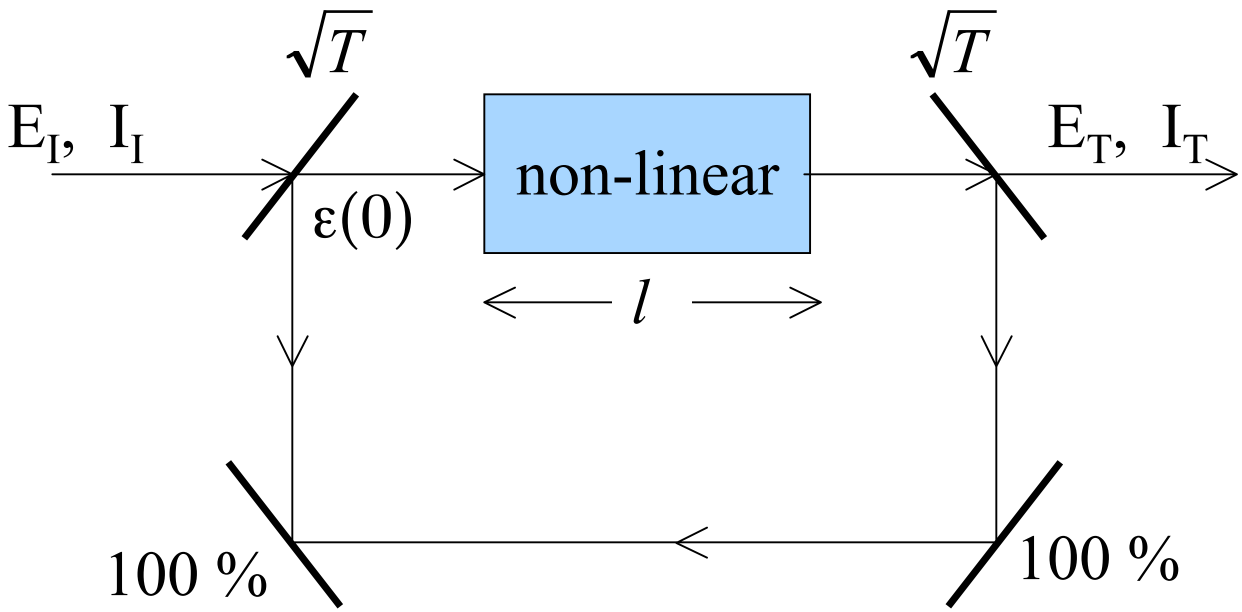
$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

where I_s is the saturation intensity

- The cavity is setup for resonance.
- At small intensities, the absorption is high and the output is low.
- As the intensity increases beyond I_s, the absorption rapidly decreases (power broadening) and the output goes to high.

The field inside the cavity comes from the addition of the drive and what is already there

$$\text{Let } \varepsilon_{n+1}(0) = \sqrt{T}E_I + \mathrm{Re}^{-\alpha l} \; e^{iKL}\varepsilon_n(0)$$



Absorptive Bistability

A saturable absorber, at resonance has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

where I_s is the saturation intensity

assuming that $\alpha_0 l \ll 1$ and $T \ll 1$ with $\alpha_0 l / T$ arbitrary:

$$\frac{E_T}{E_I} = \frac{1}{1 + \alpha l / T}$$

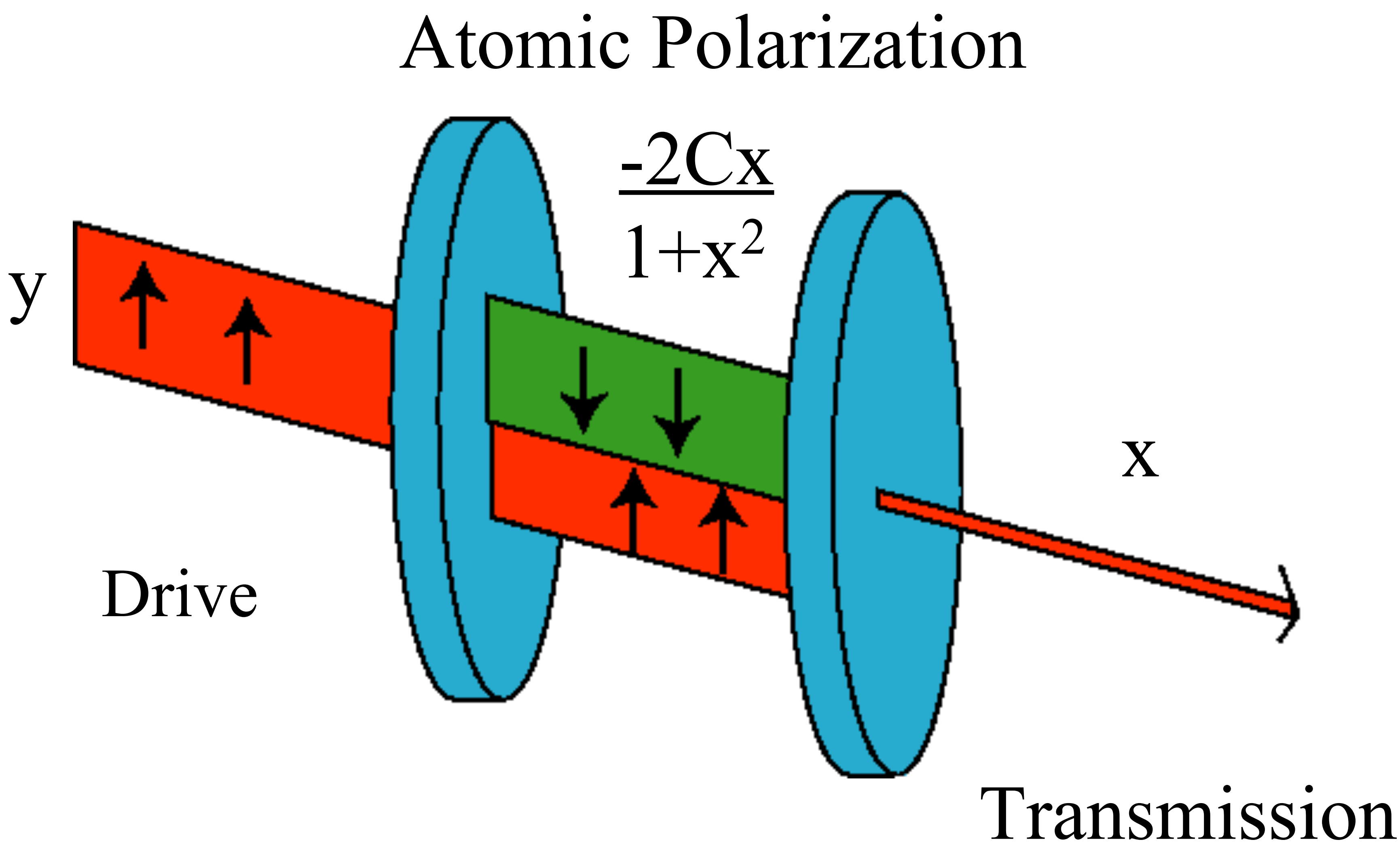
using $I = \dfrac{I_T}{T}$ leads to:

$$E_I = E_T\left[1 + \frac{\alpha_o l / T}{1 + I_T / I_s T}\right]$$

Cooperativity $\quad C = \dfrac{\alpha_o l}{T}$

with $y = \dfrac{E_I}{\sqrt{I_s}}$; and $x = \dfrac{E_T}{\sqrt{TI_s}}$:

$$y = x\left[1 + \frac{2C}{1 + x^2}\right]; \; x = y - \frac{2Cx}{1 + x^2}$$


Atomic Polarization
-2Cx
1+x²
y
Drive
x
Transmission

What do we expect on resonance for the normalized fields (x,y) and the normalized intensities to the saturation photon number (X,Y)?

For low intensity, the input and the output are linearly related,

$y = x\,(1+2C)$; $Y=X(1+2C)^2$

$$\frac{X}{Y} = \frac{1}{(1+2C)^2}$$

$y = x\,(1+2C)$

For very high intensity,
$y = x$; $Y=X+4C$

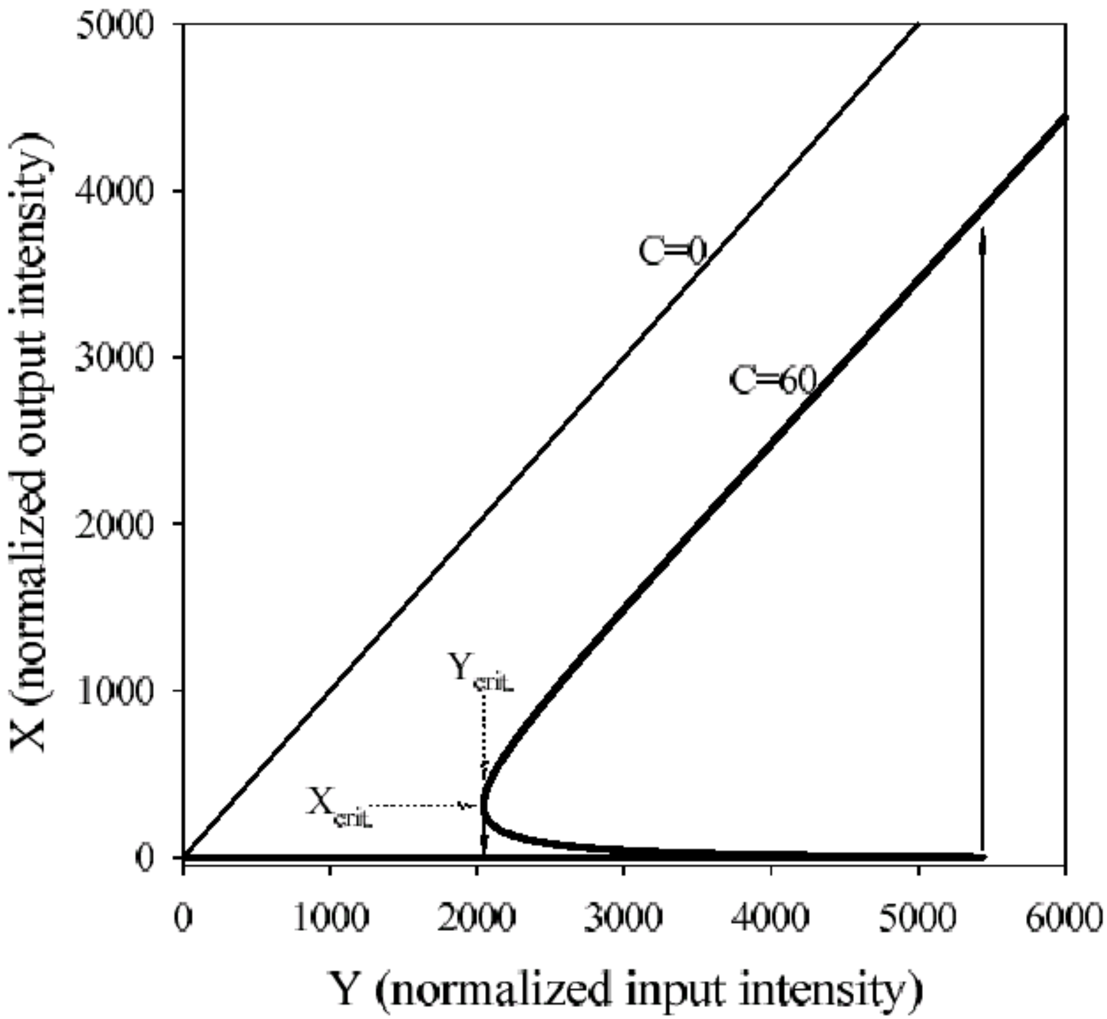
$$\frac{X}{Y} = 1 + \frac{4C}{Y}$$

$$y = x\left[1 + \frac{2C}{1 + x^2}\right]$$

At intermediate intensity, there can be saturation. It happens in this simple model for the case of C>4. C (Cooperativity) is the negative of the laser pump paramenter.

The slope of the output x as a function of input y may be zero!

Input-Output response of the atoms-cavity system for two different cooperativities C=0 is with no atoms, C=60 has plenty of atoms, with a drive that can saturate them and we recover the linear relationship with unit slope between Y and X. The hysteresis is clearly visible.



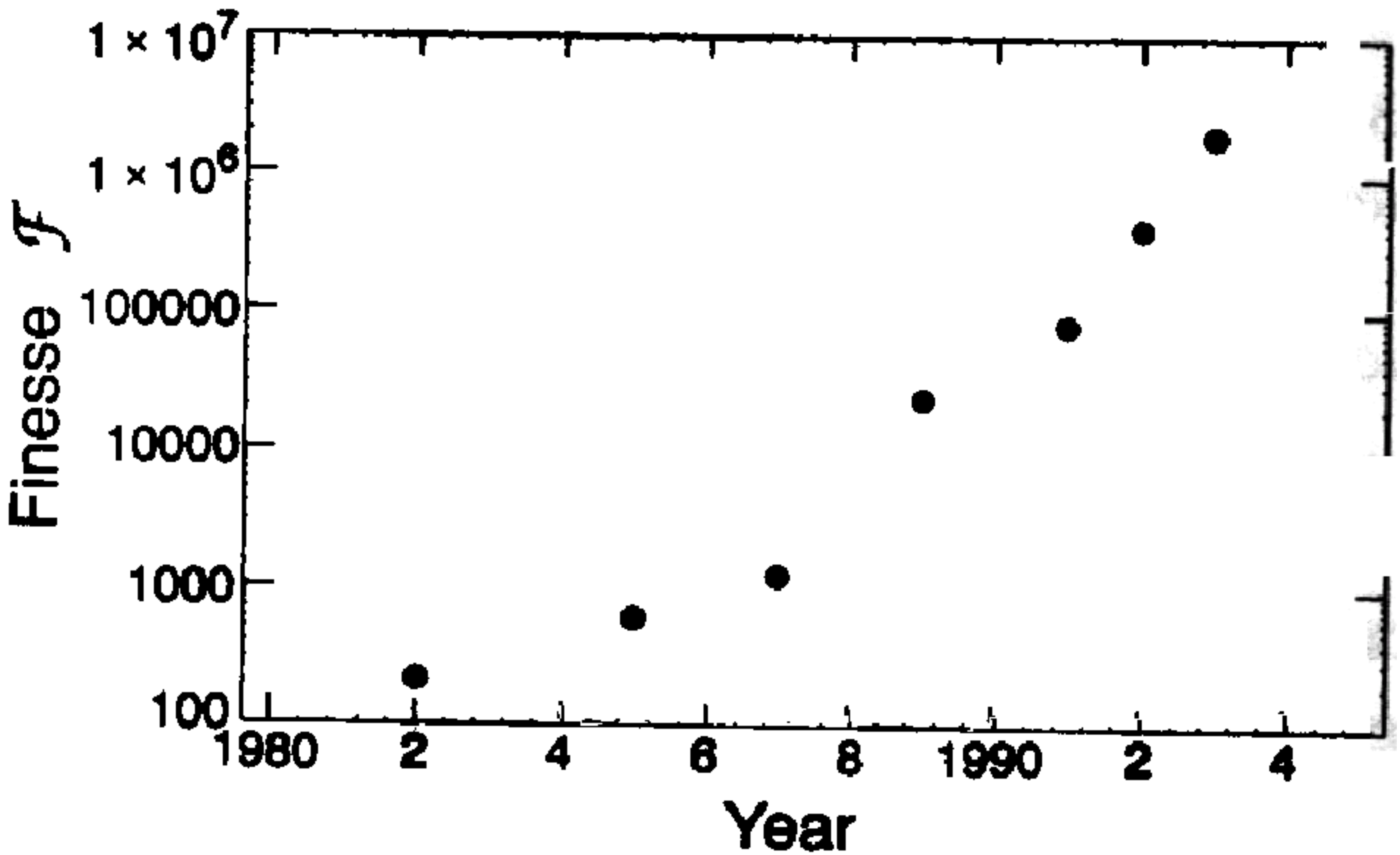
To reach the strong coupling regime in the optical regime it is necessary to make the coupling between the atoms and the cavity *g* larger than the decay avenues of the system (cavity κ) or (atoms γ).

The way to achieve this is making *g* larger, in the optical regime by making the volume smaller. For a free atom the area ratio reaches one so only remains to make R closer to unity.

However this increases linearly the decay of the cavity, as it depends on the length L of the cavity but also on the reflectivity R, keeping other losses low, such that R+T=1

$$\kappa = \frac{c}{2l}\left(1 - R\right) = \frac{c}{2l}T$$

Technological advantage to get to optical strong coupling.



Formulation of the problem: 1.- Free evolution of cavity mode and atoms, 2.- Coupling atom-cavity, 3.- Decay of atoms (reservoir), 4.- Decay of cavity field (reservoir), 5.- Drive of the cavity

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$

$$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right) \ ,$$

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$

$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$

Use this Hamiltonian to find the equations of motion of the field <a>, the atomic polarization $<\sigma^+>$, and atomic inversion $<\sigma^z>$. We assume N atoms distributed at the positions r_j in the mode of the cavity.

semiclassical decorrelation replacing $g_j\langle\hat{a}\hat{\sigma}_j^k\rangle$ by $g_j\langle\hat{a}\rangle\langle\hat{\sigma}_j^k\rangle$

Maxwell Bloch Equations are then:

Radiation field:

$$\frac{\partial}{\partial t}\langle\hat{a}\rangle = -\kappa(1+i\theta)\langle\hat{a}\rangle + \sum_{j=1}^{N} g_j\langle\hat{\sigma}_j^-\rangle + \mathcal{E},$$

Atomic polarization:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^-\rangle = -\gamma_\perp(1+i\Delta)\langle\hat{\sigma}_j^-\rangle + g_j\langle\hat{a}\rangle\langle\hat{\sigma}_j^z\rangle,$$

Atomic inversion:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^z\rangle = -\gamma_\parallel\left(\langle\hat{\sigma}_j^z\rangle + 1\right) - 2g_j\left(\langle\hat{a}\rangle\langle\hat{\sigma}_j^+\rangle + \langle\hat{a}^\dagger\rangle\langle\hat{\sigma}_j^-\rangle\right).$$

The cavity and atomic detunings θ and Δ are defined as

$$\theta = \frac{\omega_c - \omega_l}{\kappa} \quad \text{and} \quad \Delta = \frac{\omega_a - \omega_l}{\gamma_\perp}.$$

State Equation of Optical Bistability, (Cavity QED).

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

y is the normalized input field $y=E/n_0^{1/2}$, it is the field inside the cavity with no atoms.

x is the normalized output field $x=\langle a\rangle/n_0^{1/2}$, it is the field inside the cavity with atoms.

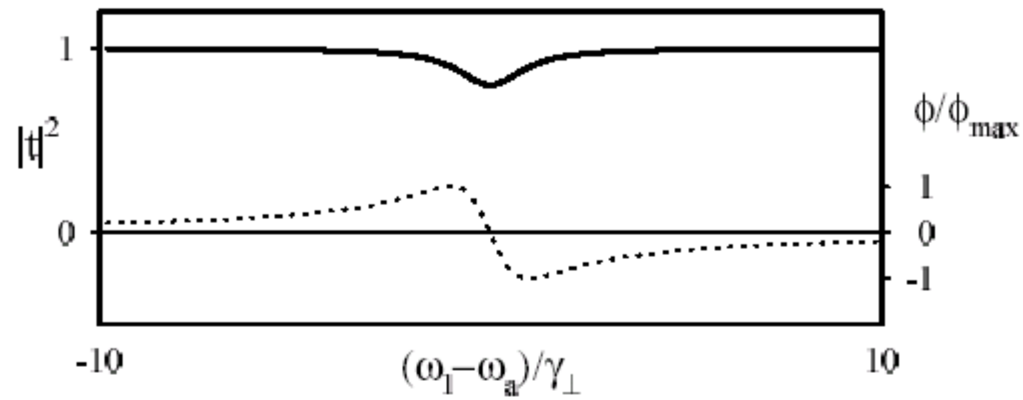
Δ is the normalized atomic detuning $\Delta=(\omega_{atom}-\omega_{laser})/\gamma/2$.

θ is the normalized cavity detuning $\theta=(\omega_{cavity}-\omega_{laser})/\kappa$.

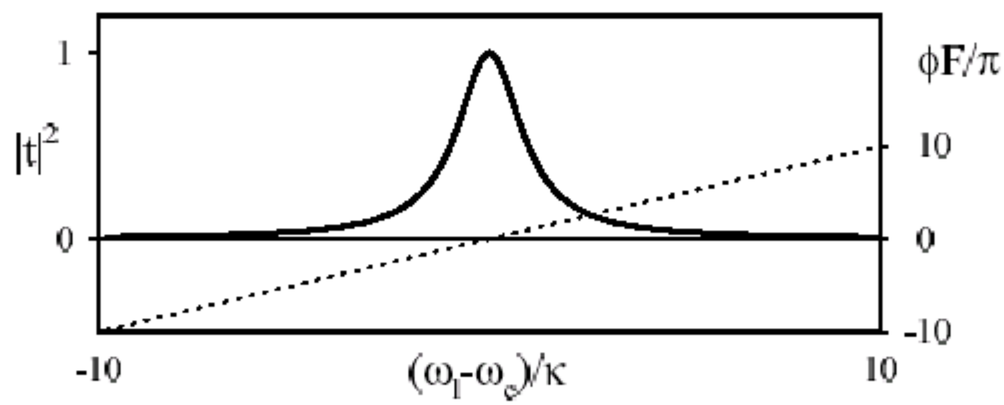
This equation makes explicit that the phase of the input and output fields need not be the same.

Constructive interference happens when the phase is zero.

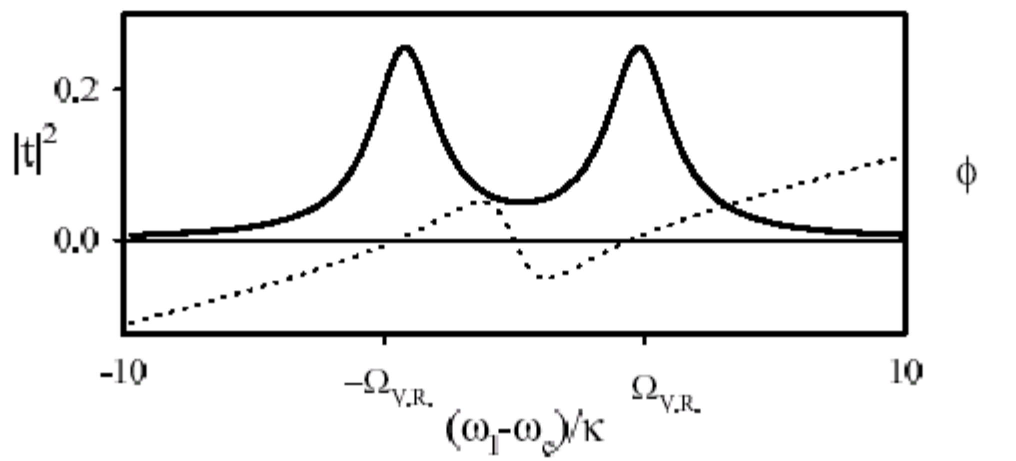
Transmission of light of different frequencies close to resonance and phase of the atoms alone (below saturation)



Transmission of light of different frequencies close to resonance and phase of the field inside for cavity alone. The phase changes by π in a free spectral range. So for high Finesse F it is linear across the resonance.



Transmission of light of different frequencies close to resonance and phase for atoms and cavity combined. Note that the peaks happen where the imaginary part crosses zero.



The transmission in the low intensity limit can be written in the following form to stress the two normal modes present:

$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} ,$$

$$A = \kappa \frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} ,$$

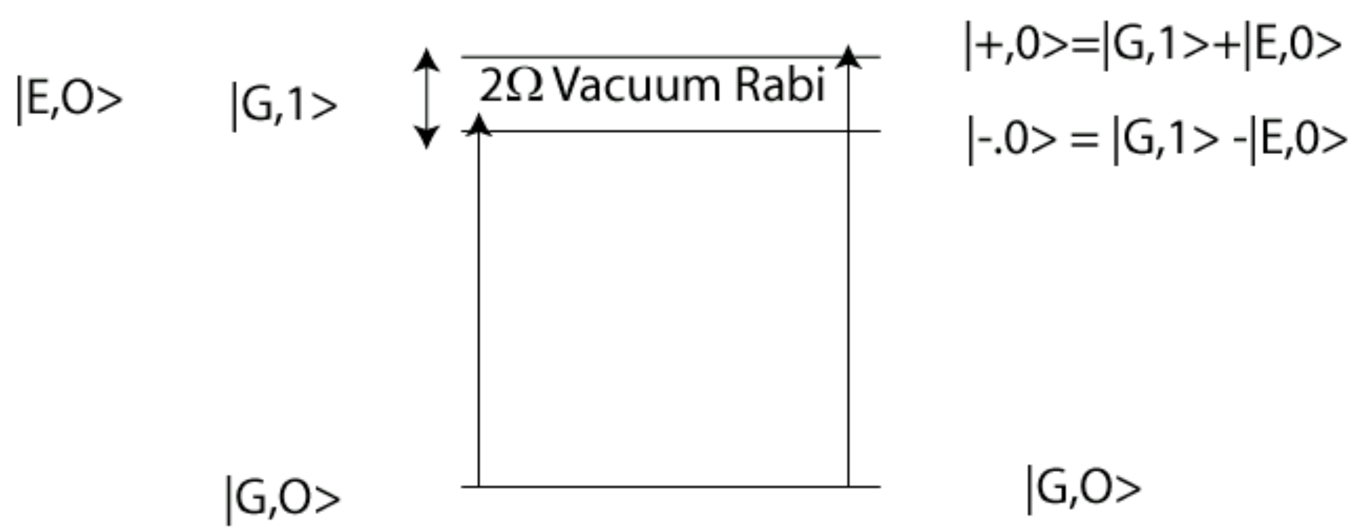
$$B = \kappa \frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} ,$$

$$\Omega_{V.R.} = g_0 \sqrt{\bar{N}}$$

$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i \sqrt{-\left(\frac{\kappa - \gamma_\perp}{2}\right)^2 + \frac{\Omega^2_{V.R.}}{1 + \gamma^2_\perp |x|^2/(\gamma^2_\perp + \Omega^2)}} .$$

We are going to probe the eigenvalue structure of the system:

The first excited state is split from the coupling between atoms and cavity so the energy levels are:



In the spectroscopy we should see two peaks corresponding to the two possible transitions between the ground state and the excited states. There is no difference using the Maxwell Bloch or the full Hamiltonian.

Some experimental considerations:

The atoms are optically pumped into the highest m sublevel of the F=3 ground state of ^{85}Rb.

The atomic beam is highly collimated and is perpendicular to the mode of the cavity.

The cavity can have a Finesse above 10^{4}.

Transmission spectroscopy of atoms-cavity system.

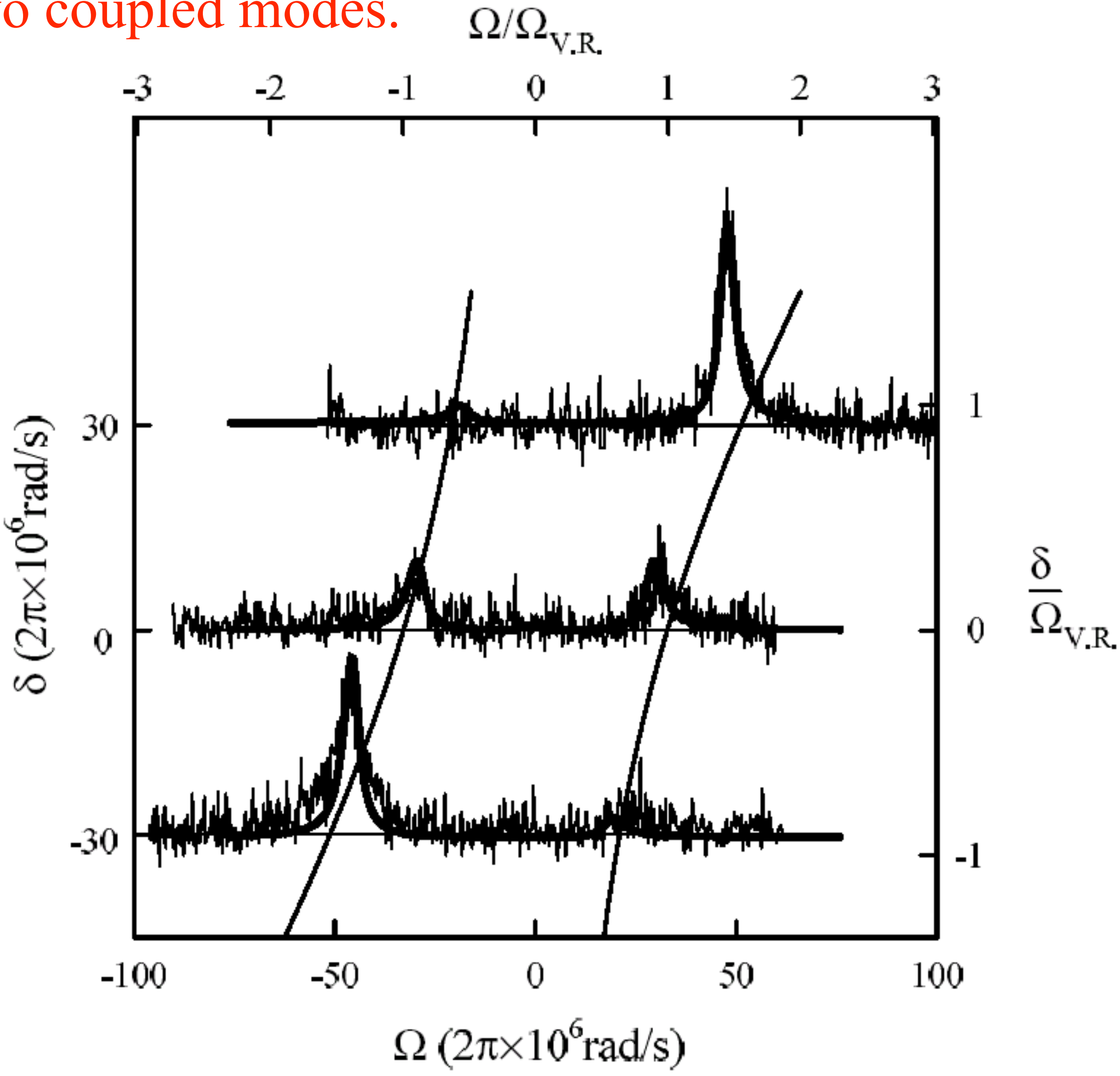
This is a way to probe the normal modes and see the eigenvalue structure of the system.

For low intensity, we have two coupled harmonic oscillators: The cavity mode and the polarization of the atoms (neglect any atomic inversion). We can observe the so-called Vacuum Rabi peaks.

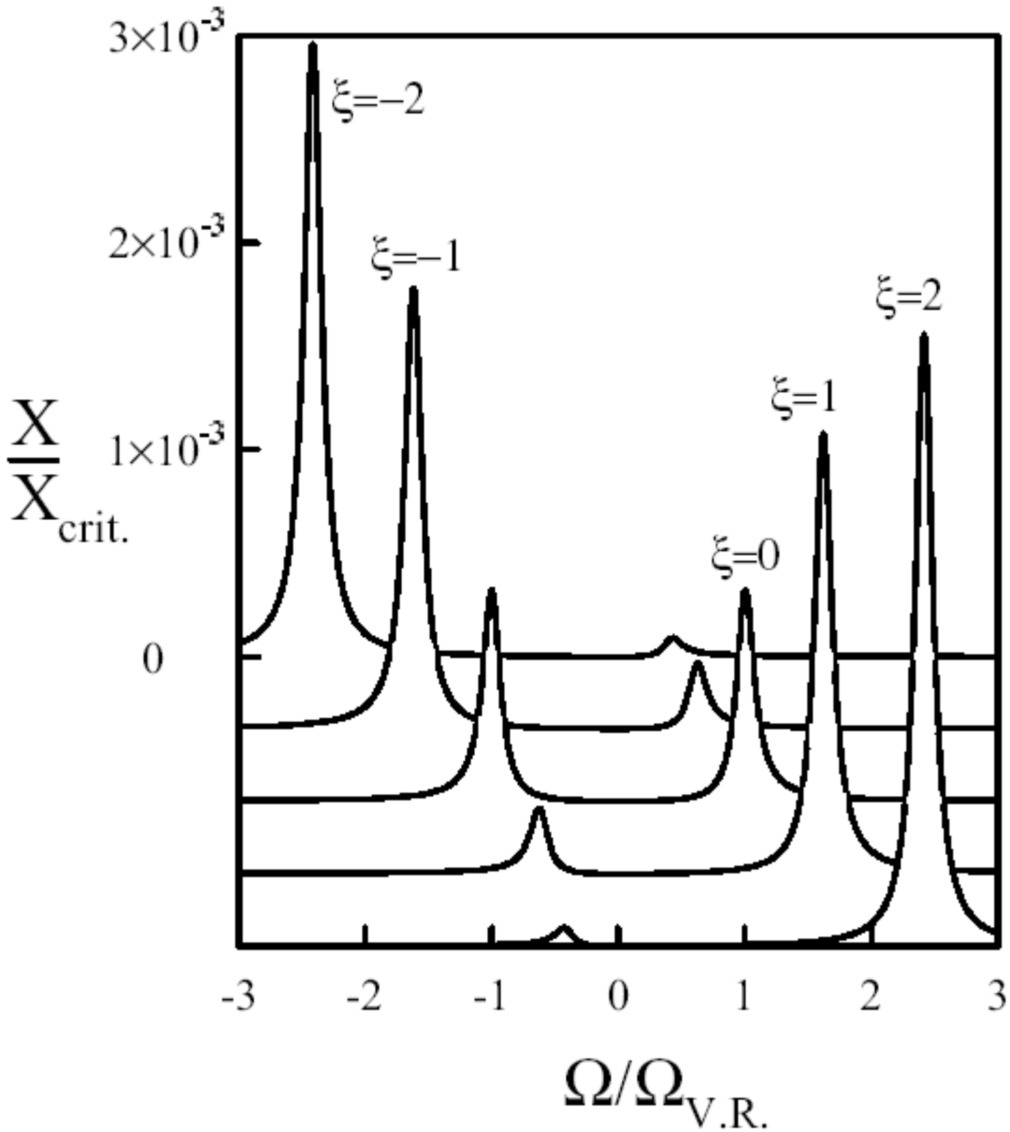
For high intensity, the atoms saturate and so one only sees the Fabry Perot fringe from the cavity.

For intermediate intensity, we have two anharmonic coupled oscillators that show frequency hysteresis.

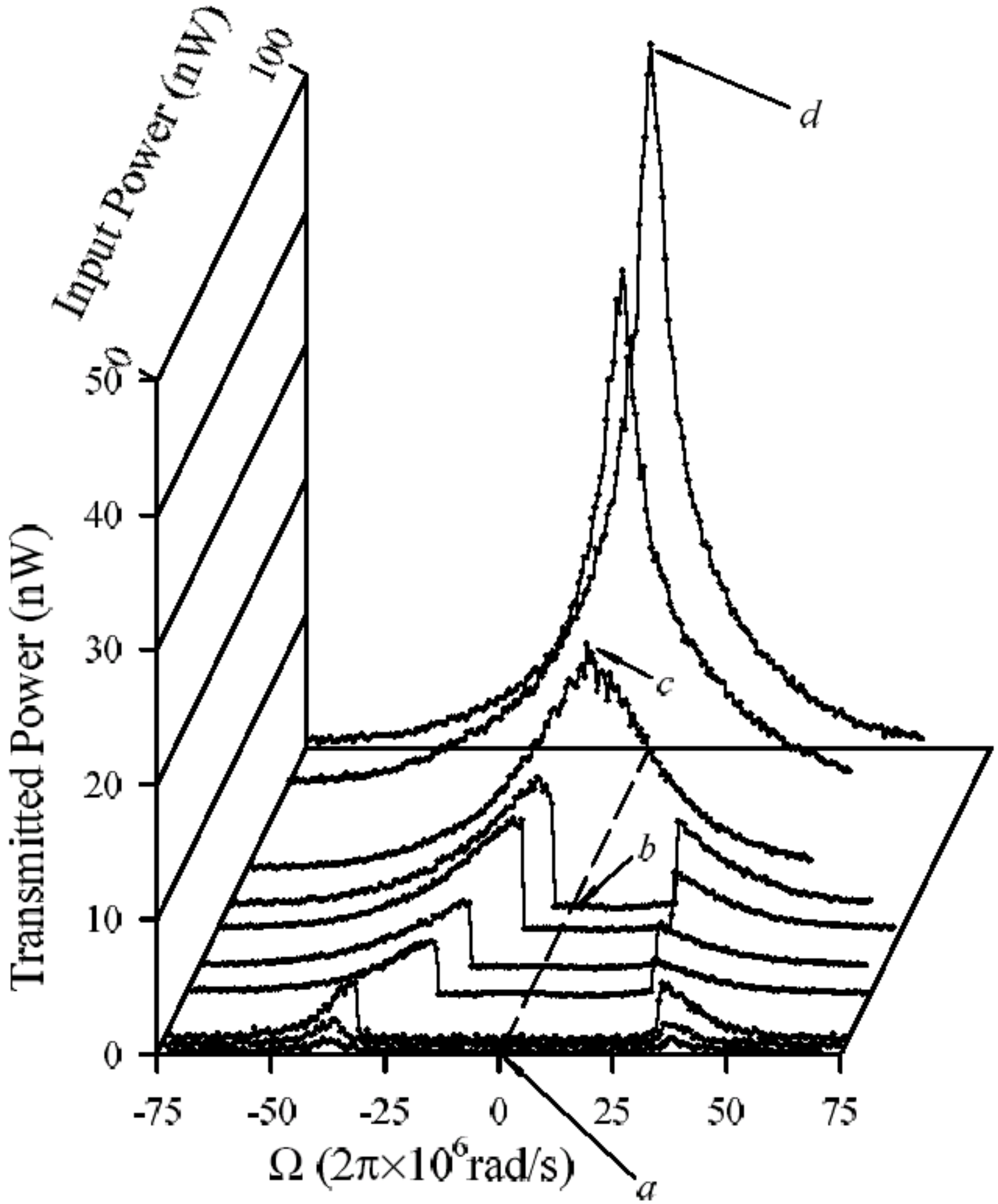
Transmission Spectra at low intensity for different atomic detunings. Note the Vacuum Rabi peaks and the "avoided crossing" of the two coupled modes.



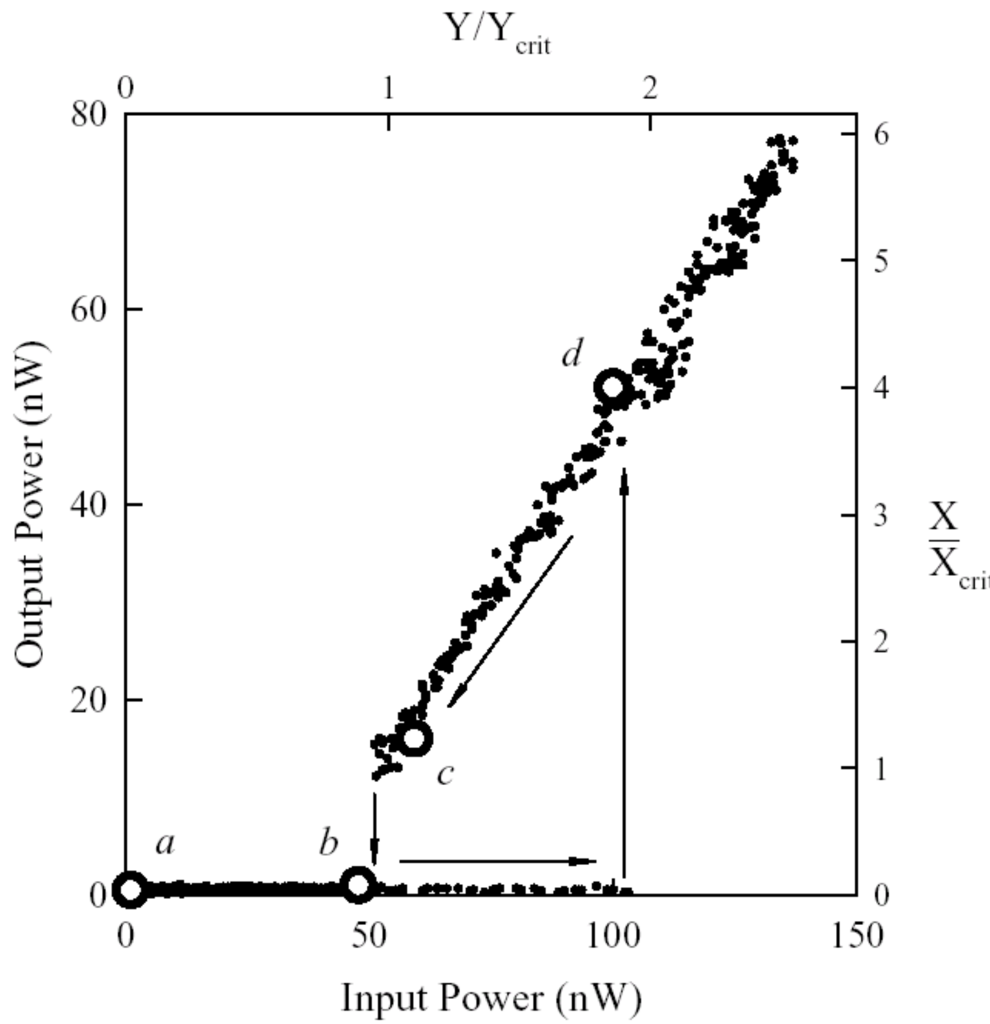
Calculation of the low intensity transmission spectra for different atomic detunings



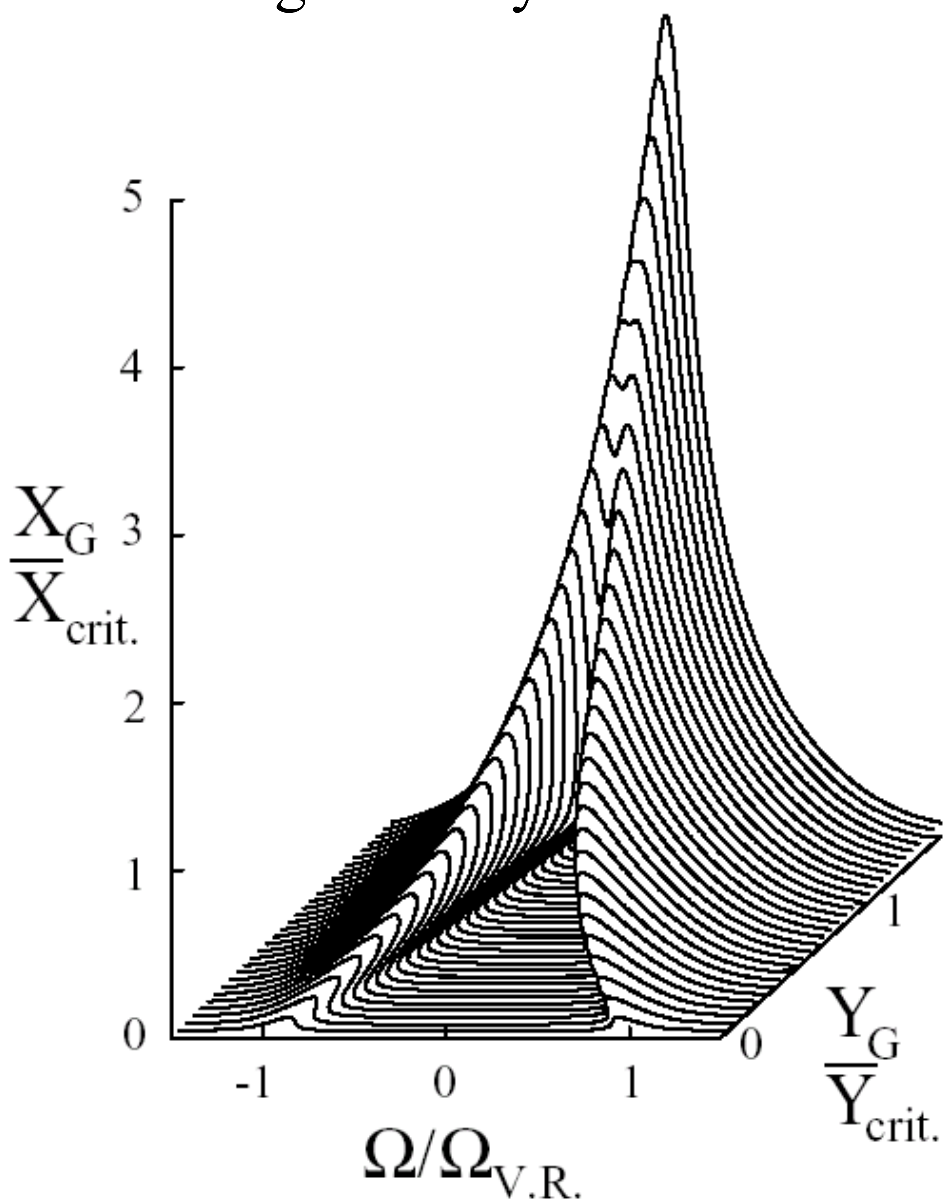
Transmission spectra for different intensities no detuning, C=78.

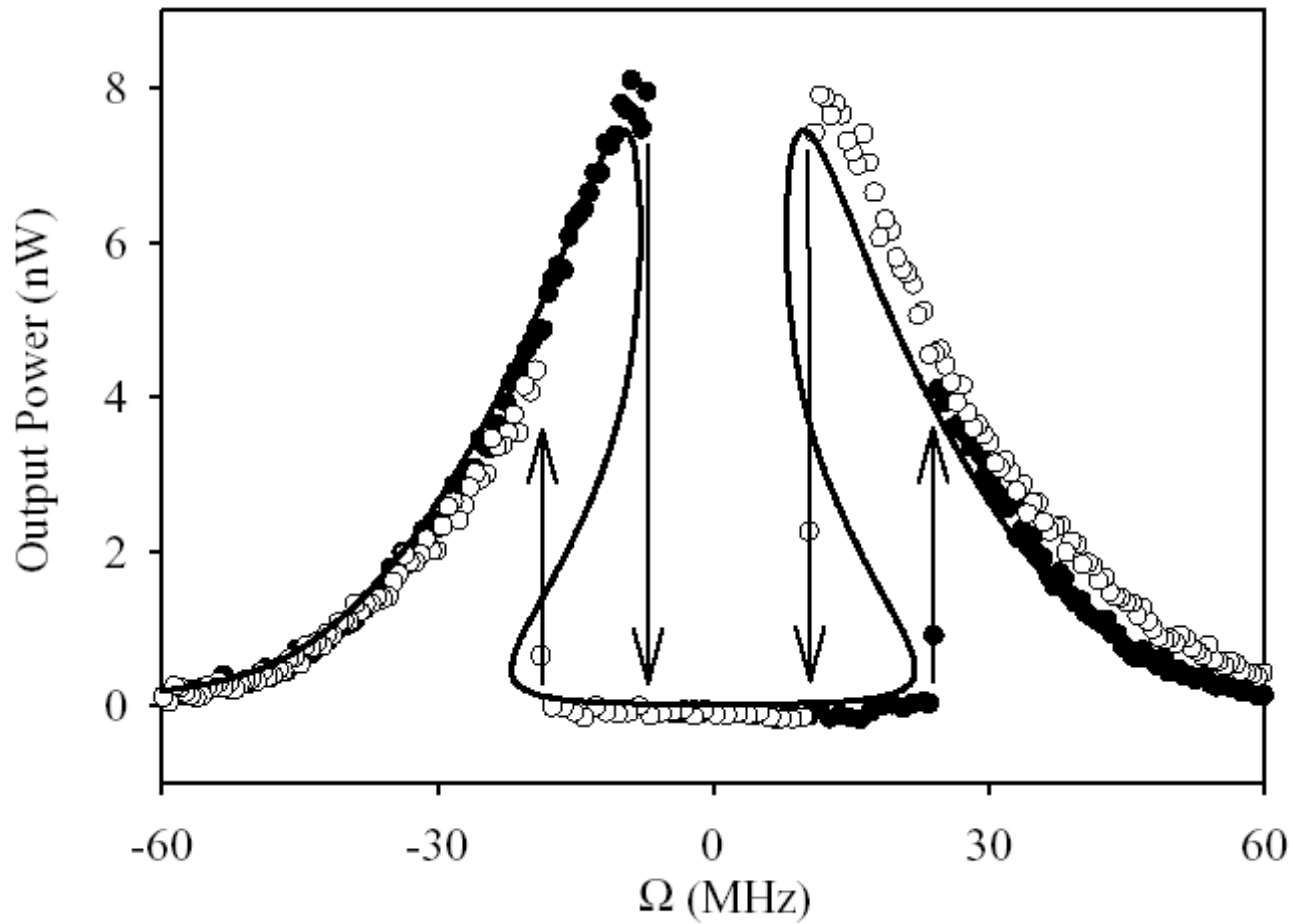


Input-Output hysteresis curve for the parameters of the transmission spectroscopy, indicating qhere the different behavior appears. (a) Vacuum Rabi peaks, (b) anharmonic oscillator, (c) broanened single peak, and (d) Fabry Perot resonance with the atoms saturated.



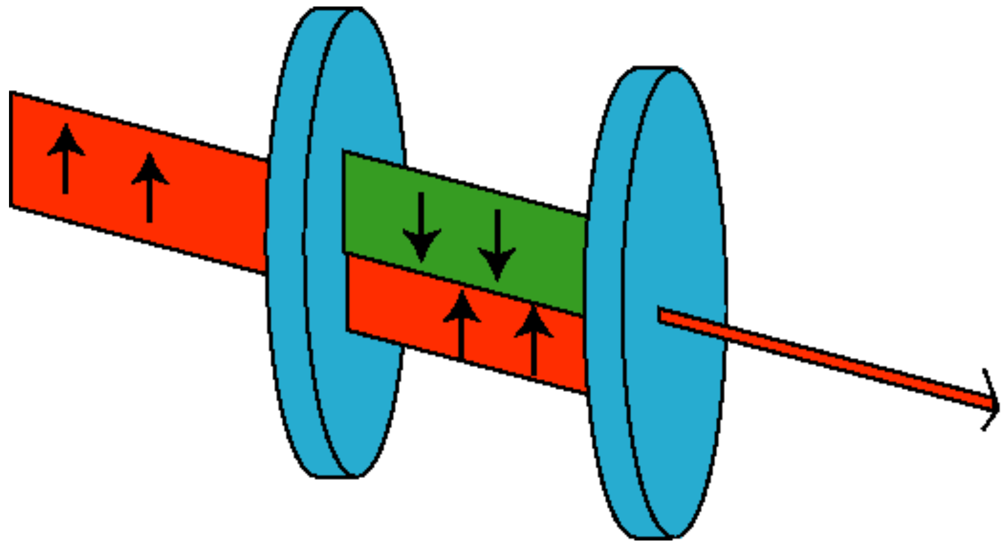
Theoretical calculation of the transmission spectra as a function of the driving intensity.





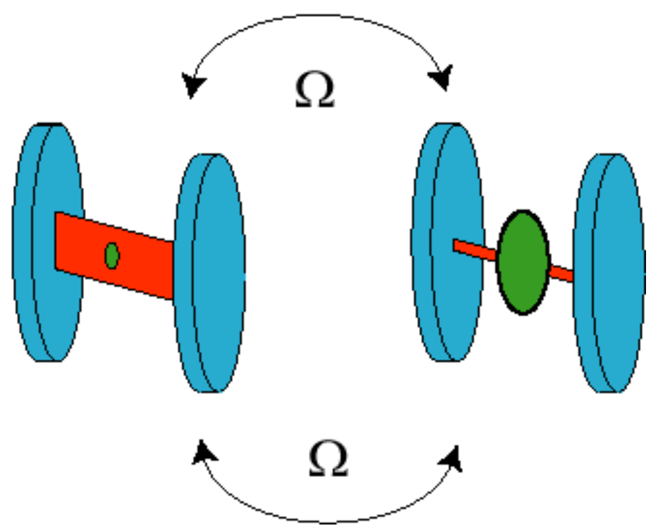
Hysteresis of the light from the coupled atoms-cavity system. Two different scans with equal input intensities are shown. Filled circles mark the scans with increasing laser frequency; open circles mark scans with decreasing laser frequency. The lines are theoretical calculations from a semiclassical theory.

Steady State:

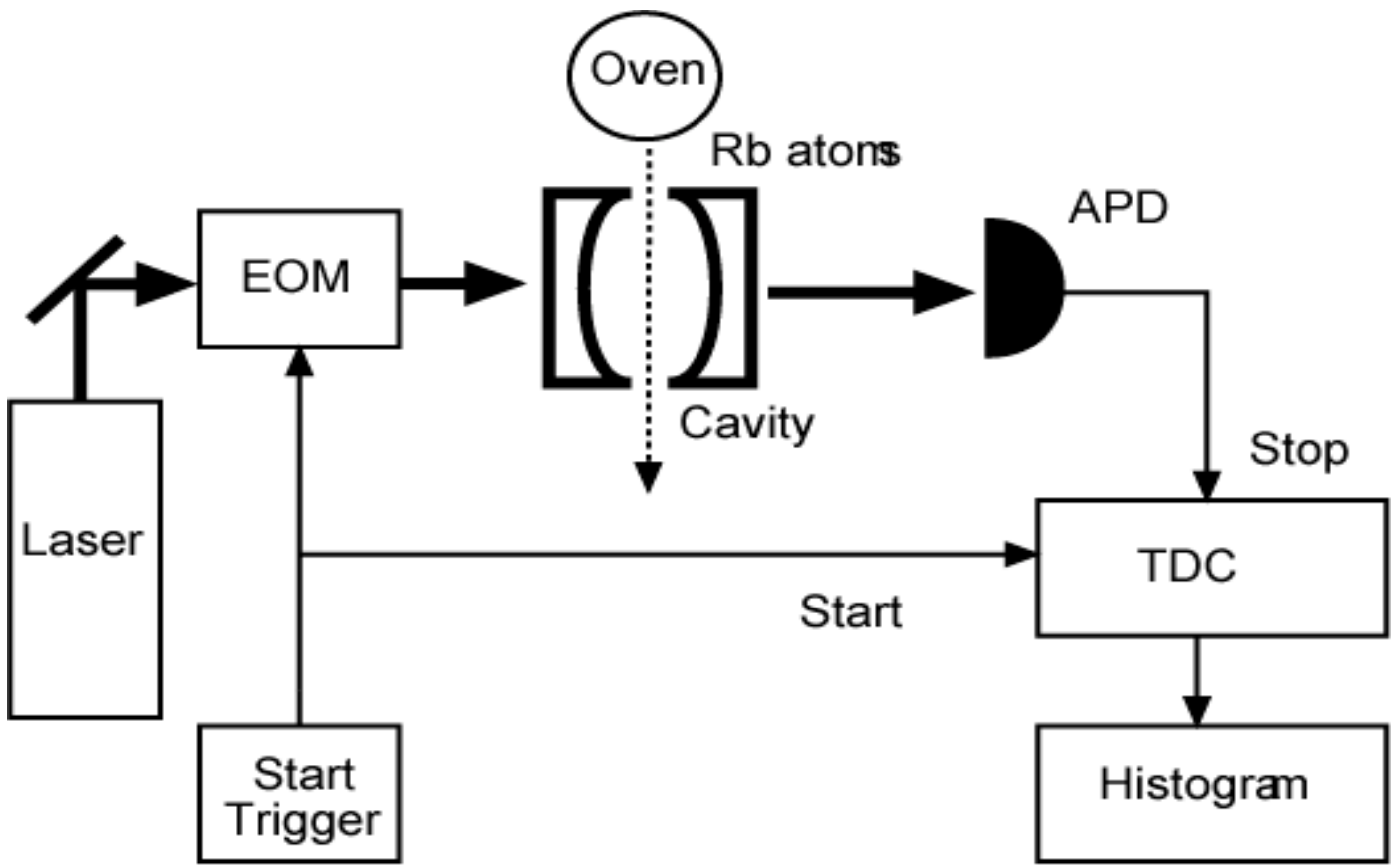


Exchange of Excitation:

Cavity Mode and Atoms



Time response



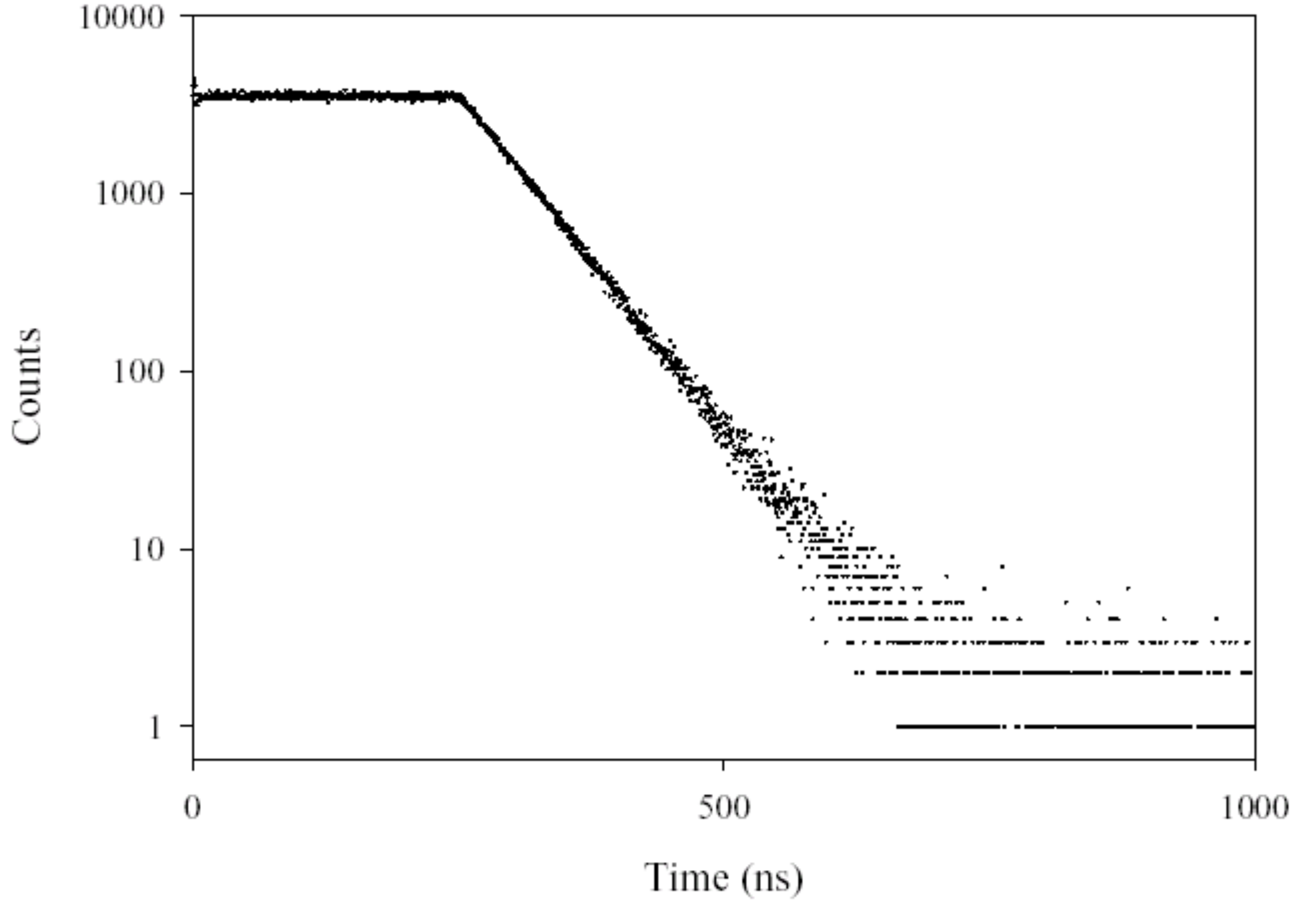
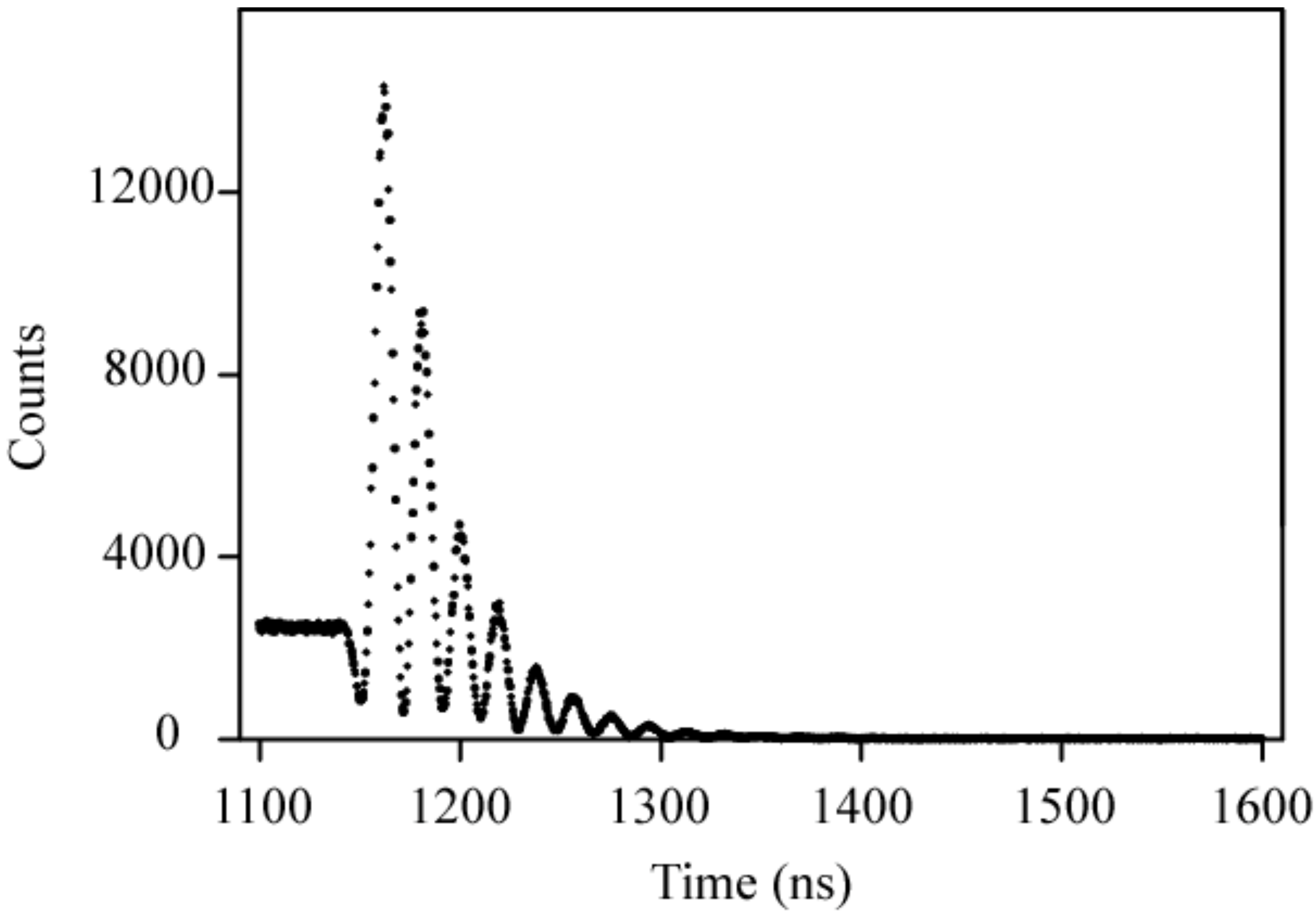
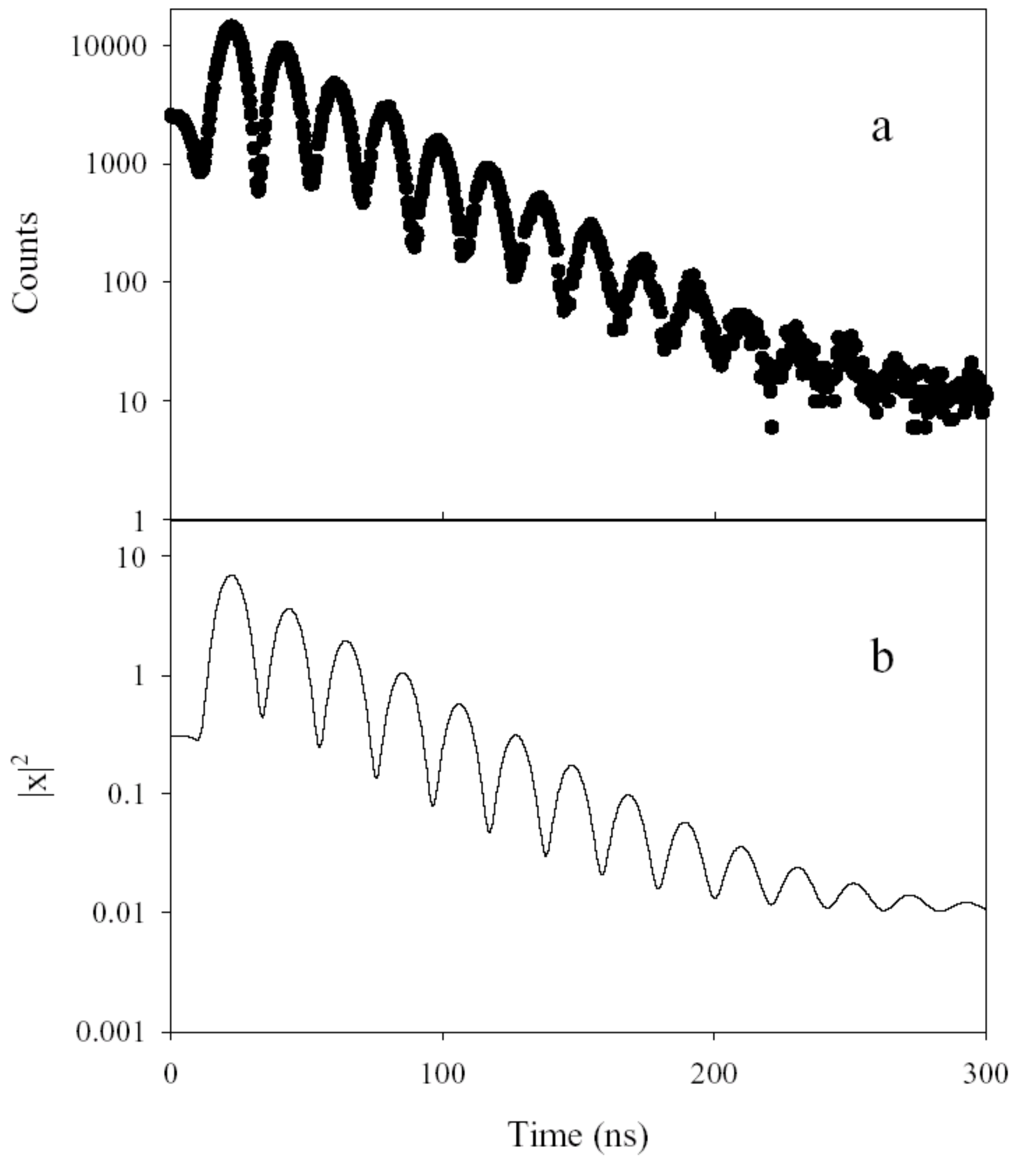


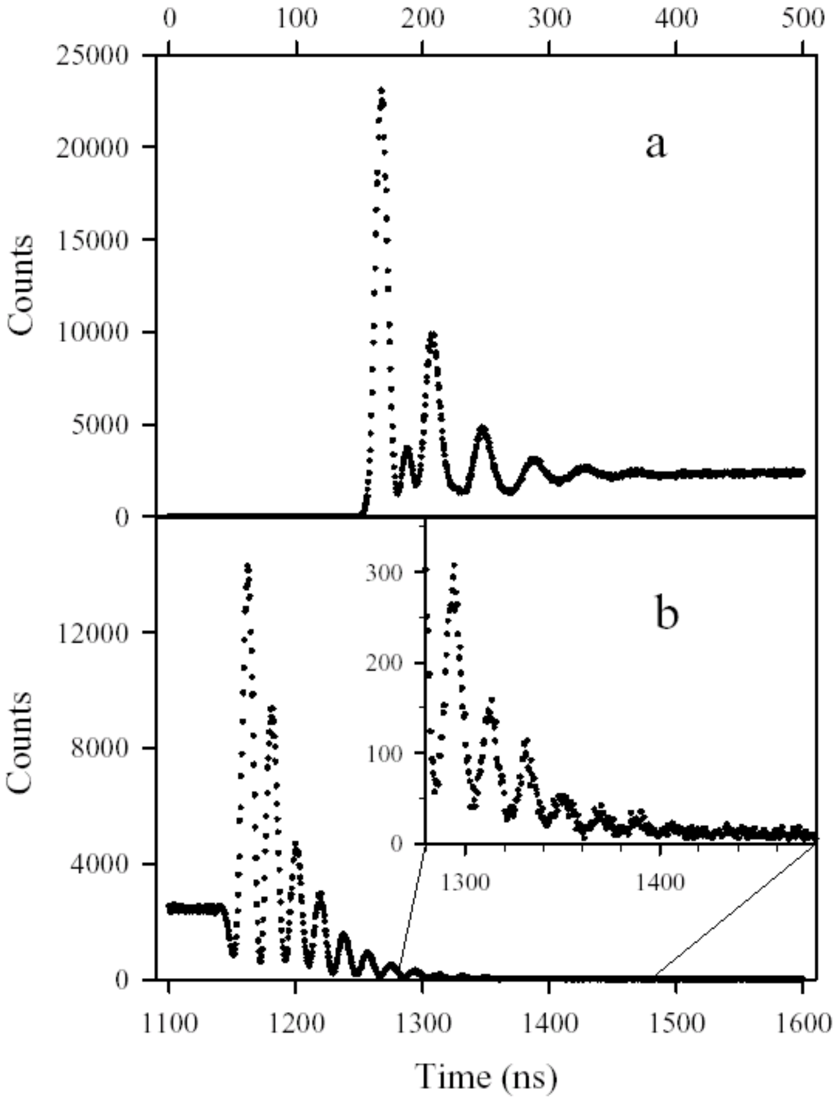
Figure 3.11: Turn-off response of an empty cavity. A line fit to the data gives a decay rate of $1.38 \pm 0.02 \times 2\pi \times 10^6$ rad/s.

Intensity response to step down of the atoms-cavity system.



Response in logarithmic scale; a) experiment, b) theory





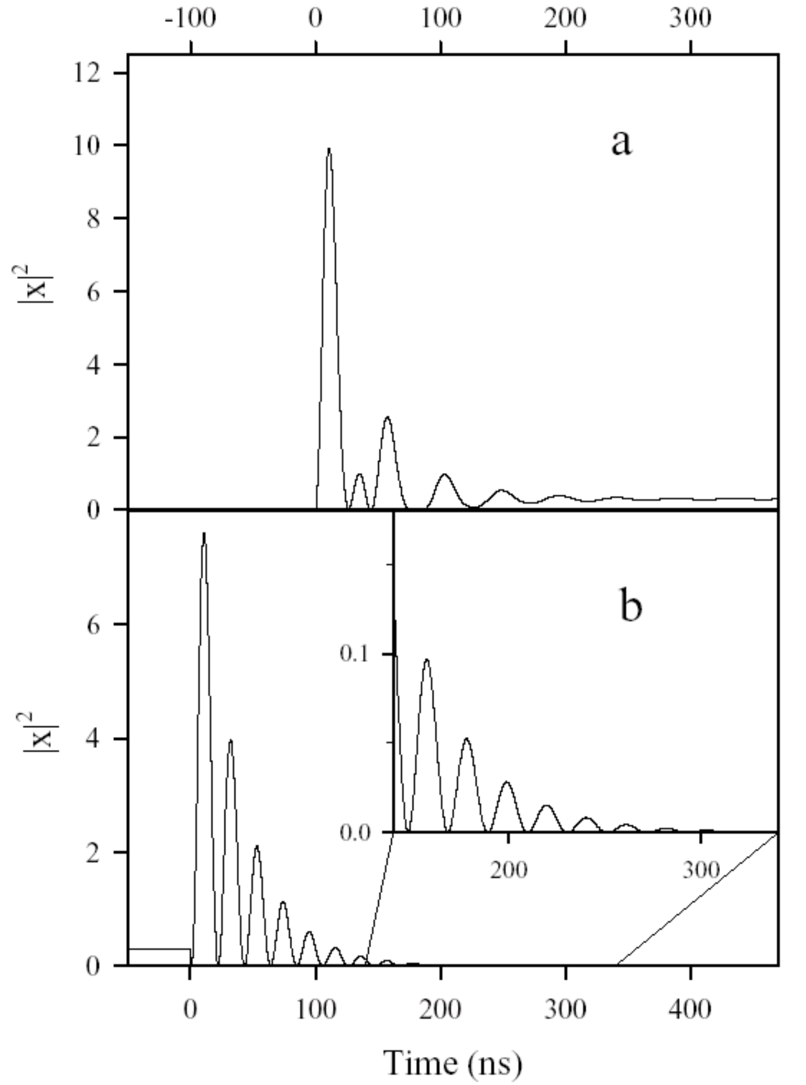
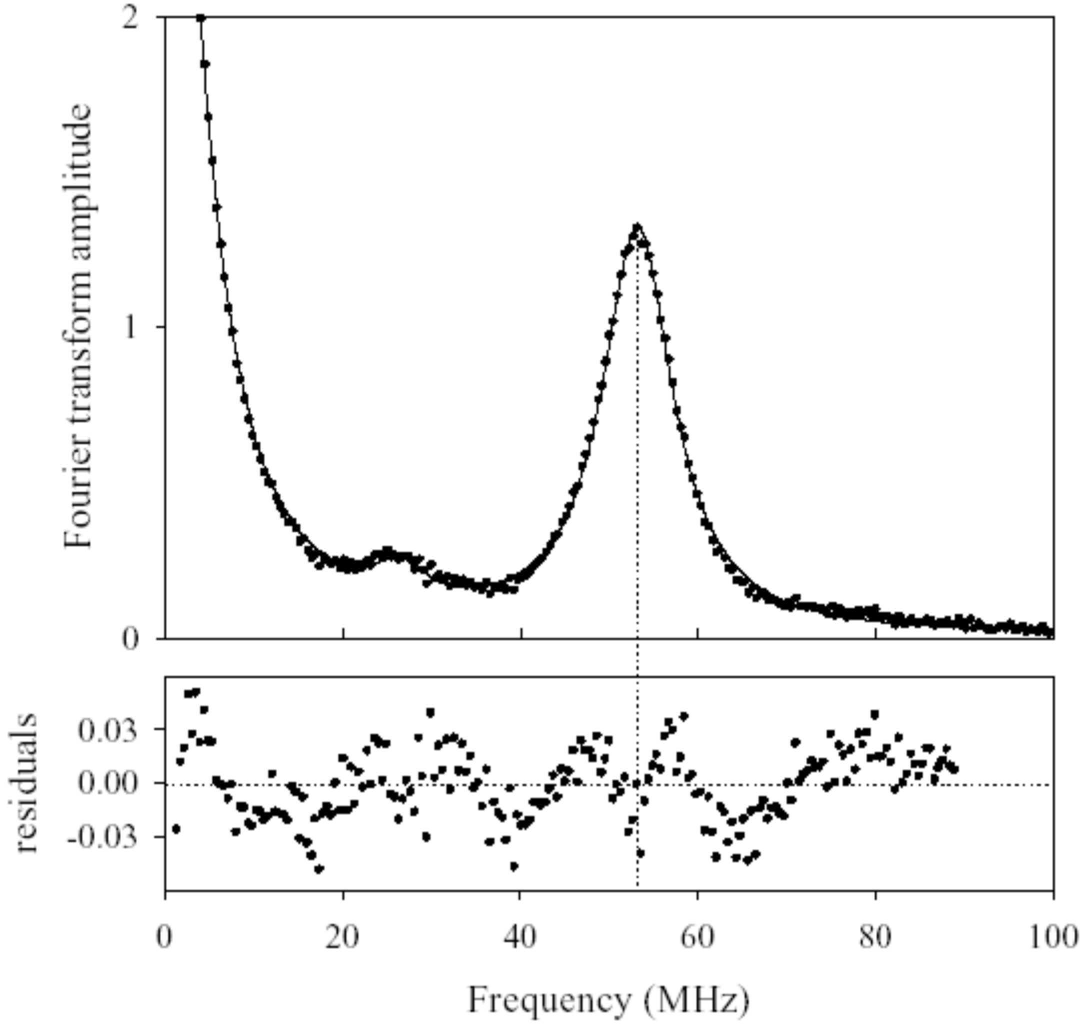


Figure 3.7: Theoretical curves from a numerical integration of Eqs 3.14–3.16 for the same parameters as in Fig. 3.6.

Fourier Transform of the Step down response

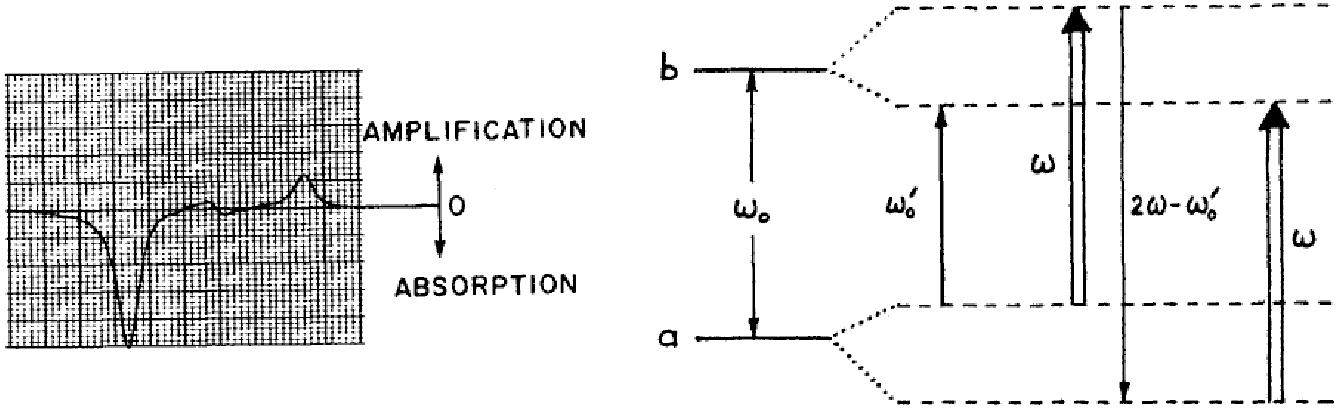


Dynamical instabilities:

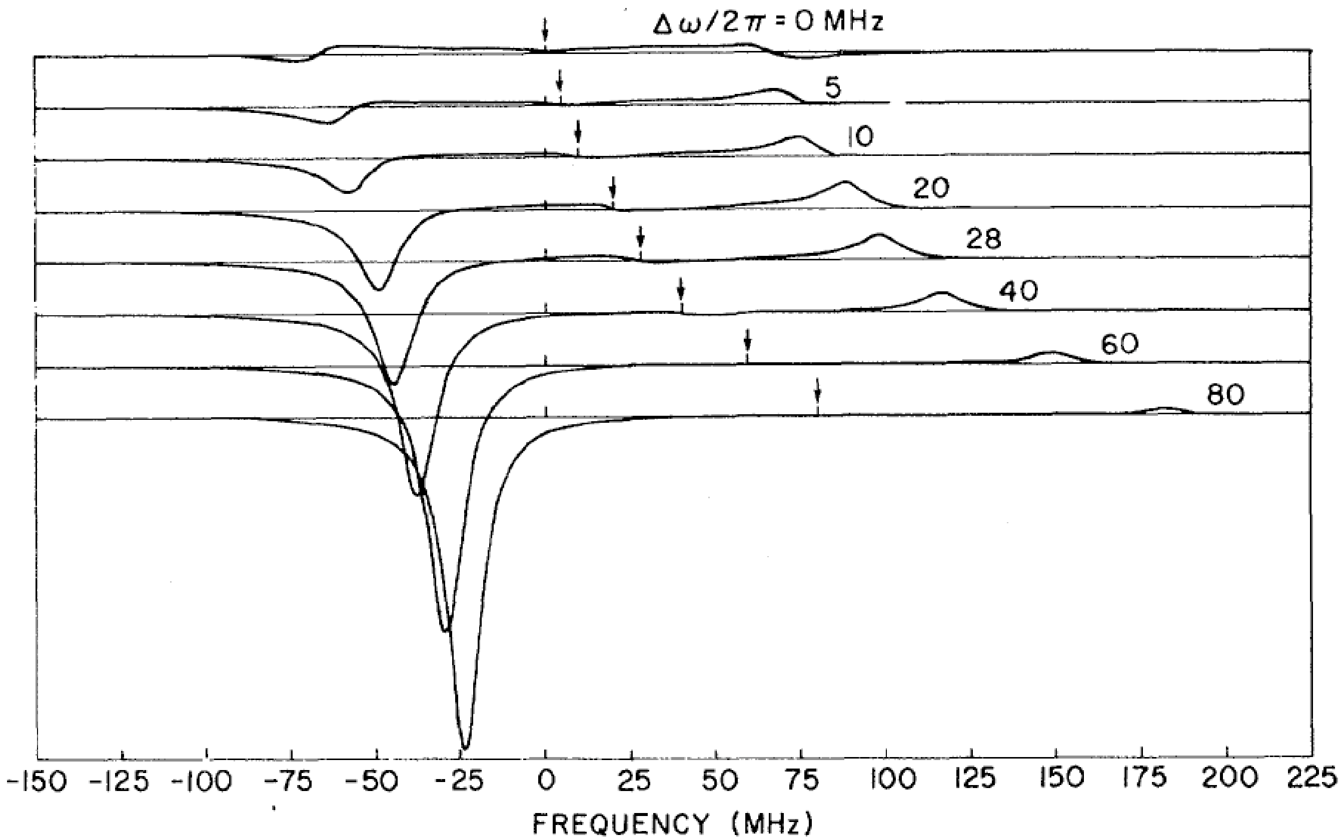
There can be gain in the system and so the output shows oscillations

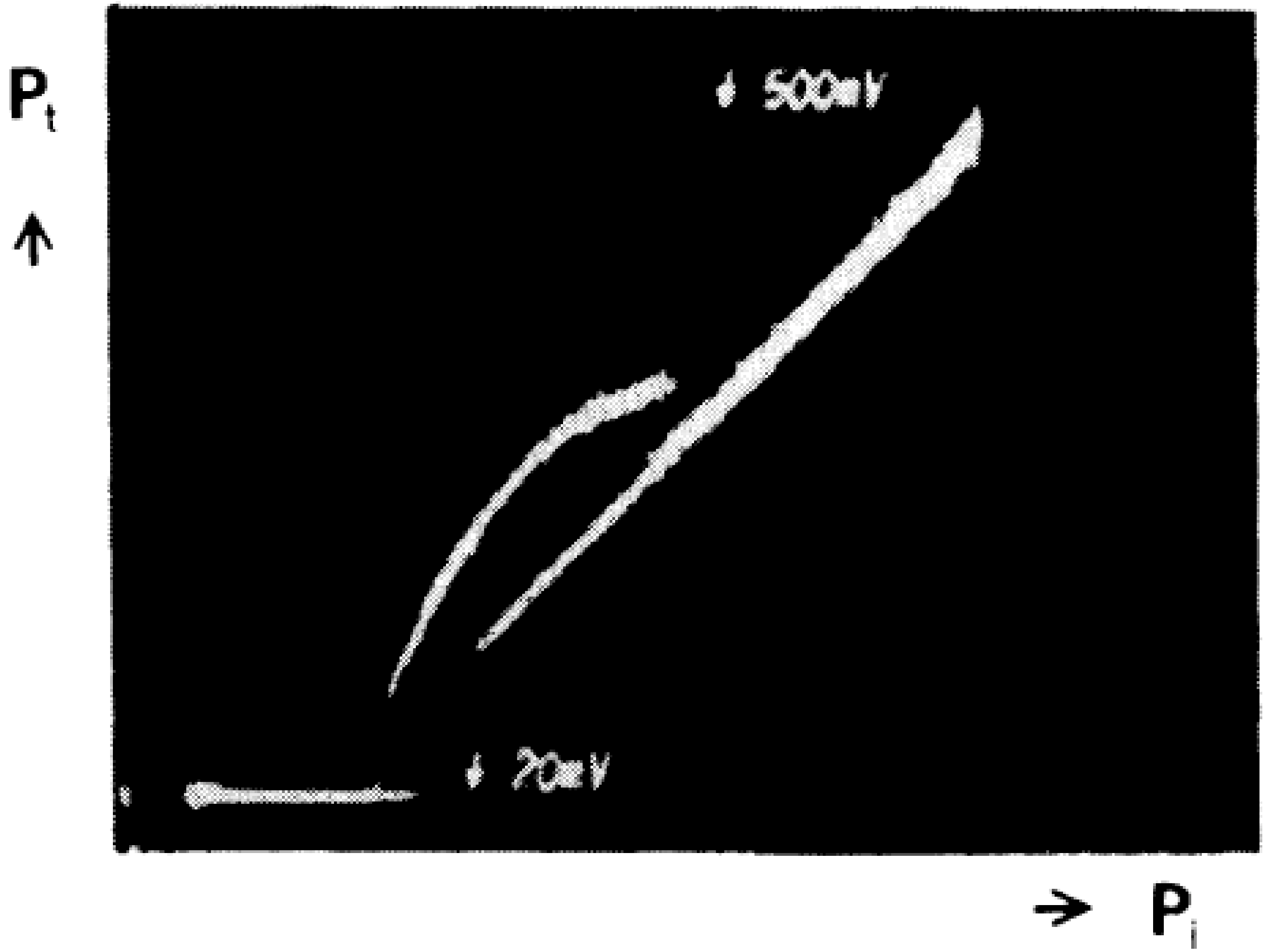
If the gain coincides with the cavity resonance then you will see oscillations.

Predictions of chaos

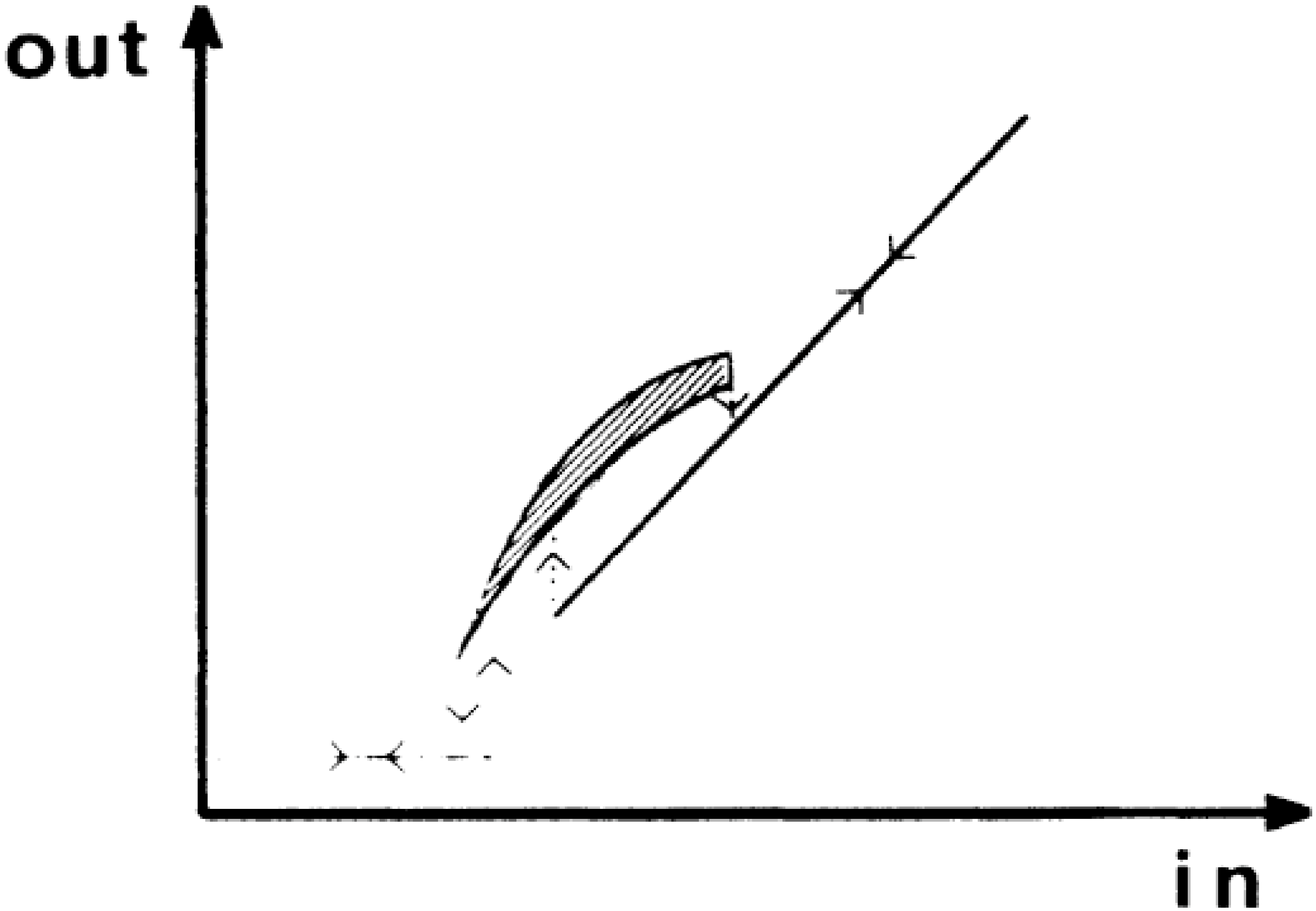

AMPLIFICATION
0
ABSORPTION
b
a
ω_o
ω_o'
ω
$2\omega - \omega_o'$
ω

(b)


$\Delta\omega/2\pi$ = 0 MHz
5
10
20
28
40
60
80
-150 -125 -100 -75 -50 -25 0 25 50 75 100 125 150 175 200 225
FREQUENCY (MHz)

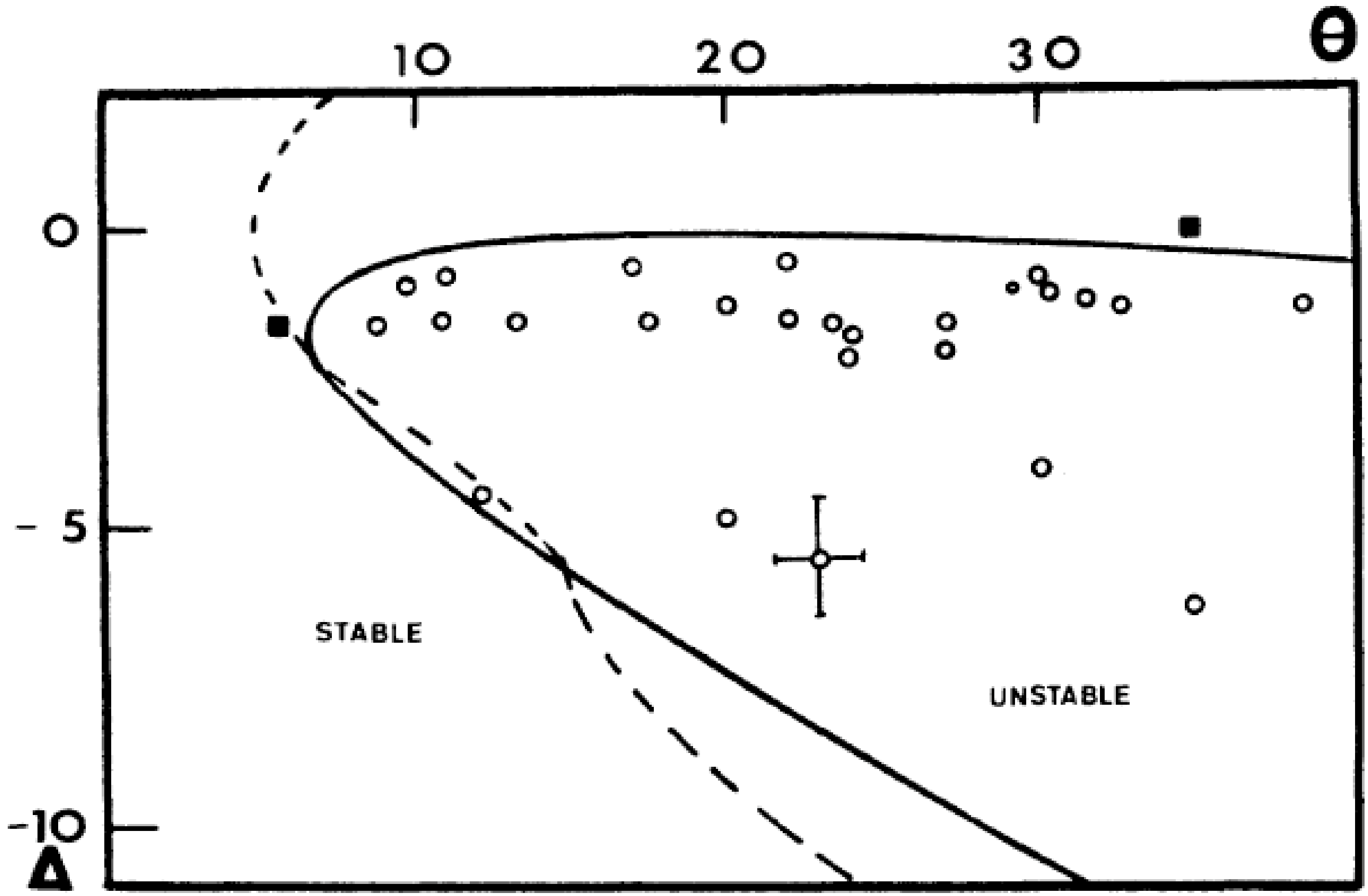


Trace of input output showing unstable region

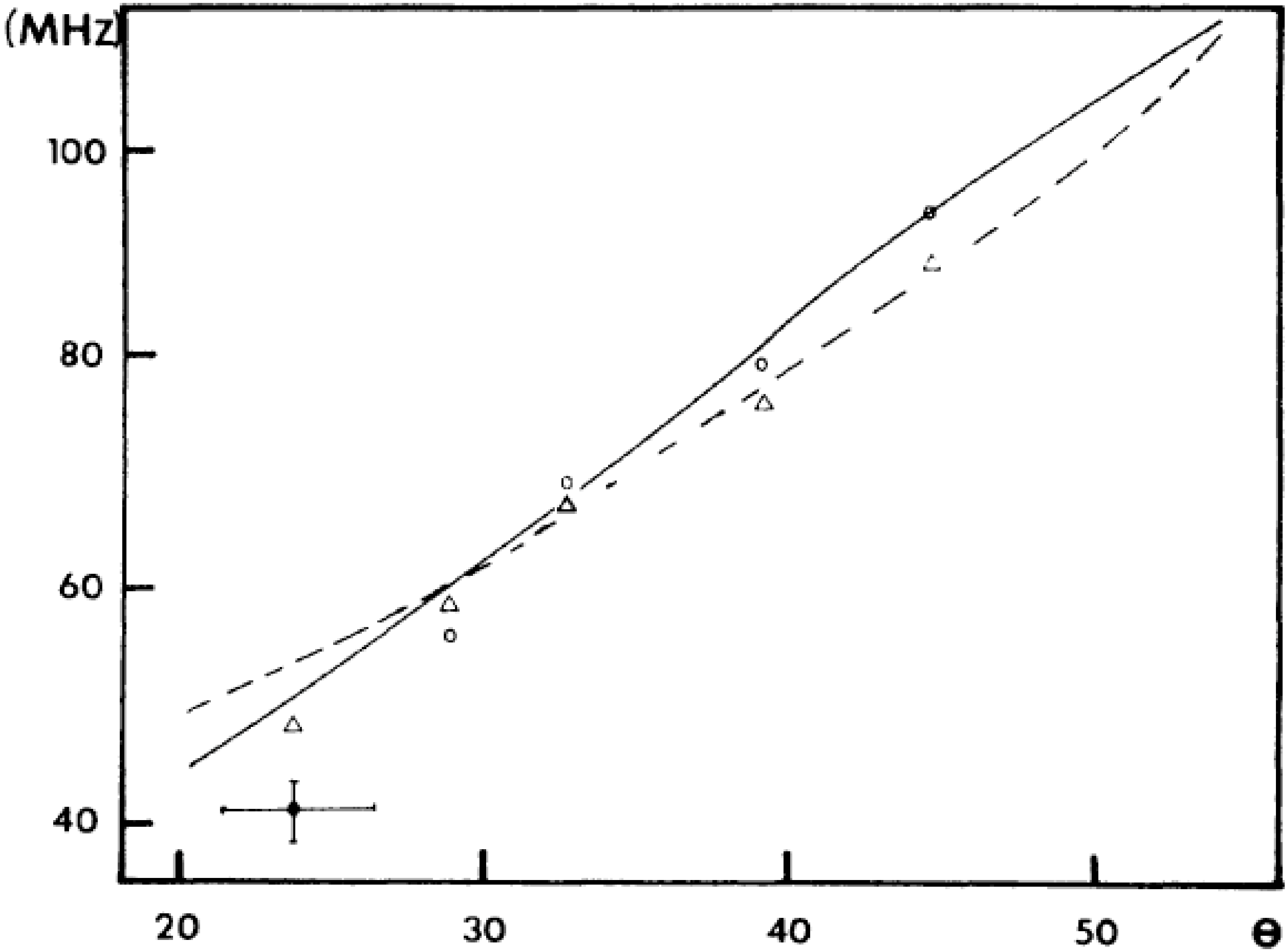


Dynamic instability at high intensity (upper branch)

Instability parameter space



Cavity detuning versus frequency of oscillation



Thanks